CALCULATION OF A TUBULAR ELECTRON BEAM IN AN AXISYMMETRIC MAGNETIC MIRROR

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A solution obtained by the method of averages [1] makes it possible to calculate a helical beam in a weakly nonuniform field. In the present paper, an approximate solution for a tubular helical beam in a weak axisymmetric magnetic mirror [3] is obtained with allowance for the space-charge field. The solution is obtained by successive approximations with respect to the ratio ε_* of the characteristic width a_* of the beam to the characteristic dimension of the external-field nonuniformity L_*, under the assumption that the beam may be represented in the form of two subfluxes with a single-valued irrotational field of the common pulse, ρ in each $\mathbf{p} = \nabla\chi_{(1,2)}$. The first approximation is obtained in the general case, and second approximations for beams with a small and with a large space charge.

1. Basic Equations. By representing the common pulse $p_{(1,2)}$ and the charge density $\rho_{(1,2)}$ of a two-flux beam in the form

$$\mathbf{p}_{(1,2)} = \nabla v \pm \nabla w, \qquad \rho_{(1,2)} = {}^1/_2\,(\rho \pm \delta) \tag{1.1}$$

the following equations may be written for a nonrelativistic axisymmetric beam in cylindrical coordinates (r, ϑ, z):

$$w_r^2 + w_z^2 = \Phi \equiv 2\eta\varphi - v_r^2 - v_z^2 - A^2, \qquad w_r v_r + w_z v_z = 0 \tag{1.2}$$

$$w_r \equiv \partial w / \partial r, \quad w_z \equiv \partial w / \partial z, \quad v_r \equiv \partial v / \partial r, \quad v_z \equiv \partial v / \partial z$$

$$\left\{\frac{1}{r}\frac{\partial}{\partial r} r w_r + \frac{\partial}{\partial z} w_z\right\}(\rho, \delta) + \left\{\frac{1}{r}\frac{\partial}{\partial r} r v_r + \frac{\partial}{\partial z} v_z\right\}(\delta, \rho) = 0 \tag{1.3}$$

$$\Delta\varphi = 4\pi\rho, \quad \Delta A = \frac{A}{r^2}, \quad \Delta \equiv \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{1.4}$$

Here, $\eta > 0$ is the ratio of the charge to the electron mass, φ the electric field potential, $\rho > 0$ the total charge density of the beam, $(c/\eta)rA$ the sole azimuthal component of the magnetic field potential, which is laid off from the cathode (on the cathode surface (K), according to our assumptions, $A_K = 0$); c is the speed of light. The equations (1.2) are equivalent to two energy integrals, while (1.3) is equivalent to two continuity equations written in accordance with (1.1) for the first and second subfluxes.

1.1°. The small parameter ε_* of the problem may be explicitly defined by converting (1.2)-(1.4) to a system of coordinates s, ϑ, l which is coupled with the beam

$$r = R(l) + sZ', \qquad z = Z(l) - sR', \qquad Z' \equiv dZ/dR, \; R' \equiv dR/dl \tag{1.5}$$

where l is the arc length along ϑ =const on the axial surface (s=0 of the axis); the surfaces l =const are cones orthogonal to the surface s=const. The metric of the system s, ϑ, l is defined as follows [2]:

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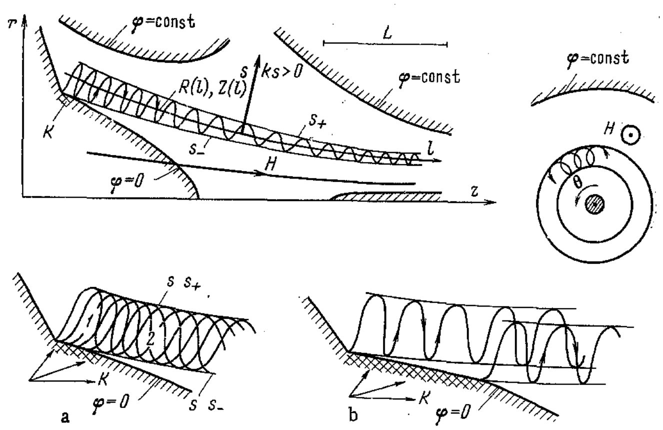

Fig. 1

$$dr^2 + r^2 d\vartheta^2 + dz^2 = ds^2 + (R + sZ')^2\, d\vartheta^2 + (1 - ks)^2\, dl^2\ ,\ k \equiv R'' / Z' \tag{1.6}$$

Here, $k(l)$ is the curvature of the axis in the meridional plane. Assume that the axial surface is sufficiently smooth and that it is situated inside the beam in such a way that $s_+ \leqslant a_*$ is the expression for the outer boundary, and $s_-\ (l) \geqslant -a_*$ the expression for the inner boundary of a tubular beam (Fig. 1).

Written in s, ϑ, l coordinates, Eqs. (1.2)-(1.4) have the form

$$w_s^2 + \varepsilon^2 \frac{w_l^2}{\sigma^2} = \Phi \equiv 2\eta\varphi - v_s^2 - \frac{v_l^2}{\sigma^2} - A^2, \quad v_s = -\varepsilon \frac{v_l w_l}{\sigma^2 w_s} \tag{1.7}$$

$$w = \int\limits_{s_-}^{s} w_s ds + w_-,\ w_l \equiv \frac{\partial}{\partial l} w, \quad v_l = V + \varepsilon \frac{\partial}{\partial l} \int\limits_0^s v_s ds \tag{1.8}$$

$$\nu\sigma (\rho w_s + \delta v_s) = I - \varepsilon \frac{Q'}{R}, \quad \nu\sigma (\delta w_s + \rho v_s) = \Sigma - \frac{\varepsilon J'}{2\pi R} \tag{1.9}$$

$$Q \equiv \int\limits_{s_-}^{s} (\delta v_l + \varepsilon\rho w_l) \frac{\nu}{\sigma} R ds, \quad J \equiv 2\pi \int\limits_{s_-}^{s} (\rho v_l + \varepsilon\delta w_l) \frac{\nu}{\sigma} R ds \tag{1.10}$$

$$\Delta\varphi = 4\pi\rho, \quad \Delta A = \varepsilon^2 \frac{A}{r^2}, \quad \sigma \equiv 1 - \varepsilon ks, \quad \nu \equiv 1 + \varepsilon k_z s \tag{1.11}$$

$$\Delta \equiv \frac{1}{\nu\sigma} \left\{ \frac{\partial}{\partial s} \nu\sigma \frac{\partial}{\partial s} + \frac{\varepsilon^2}{R} \frac{\partial}{\partial l} R \frac{\nu}{\sigma} \frac{\partial}{\partial l} \right\}, \quad k_z \equiv \frac{Z'}{R}, \quad Q' \equiv \frac{\partial Q}{\partial l} \tag{1.12}$$

Here, and in the following, a prime denotes the derivative of l for a fixed s. Integration over s has been performed in the continuity equations (1.9), on account of which, I, Σ, as well as V, ω, are arbitrary functions of l.

In the system (1.7)-(1.12), the smallness sign ε is put in those places where the parameter ε_* appears as a result of the transition to dimensionless quantities.

$$s / a_*, \quad l / L_*, \quad kL_*, \quad R / L_*, \quad \varepsilon_* \equiv a_* / L_* \tag{1.13}$$

Thus, we arrive at the case of a tubular beam with a large inner radius R, which moves in a magnetic field with a large-scale nonuniformity L_*

1.2°. At the boundaries of the beam, the conditions under which the oscillation rate ω_s, ω_l, which distinguishes the subfluxes, vanishes can be written as

$$w_{-}=0, \quad w_{+}=w_{*}, \quad \Phi_{-}=\Phi_{+}=0 \tag{1.14}$$

and the discontinuity conditions for the total current of the beam as

$$\varepsilon J_{+}'=2\pi R\Sigma, \quad 0<l<l_{*}; \quad J_{+}=J_{*}, \quad \Sigma=0, \quad l>l_{*} \tag{1.15}$$

Here, and in the following, +, - denote values at the outer and inner boundary, respectively; an asterisk denotes the constants of the problem. Function Σ defines the sources at the inner boundary of the beam; this boundary, according to our assumptions, coincides within the region $0<l<l_*$ with the cathode surface (K). As can be readily seen from (1.9), Σ is equal to $(v\sigma j)_K$, where j is the normal component of the emission-current density.

Within the framework of a two-flux approximation, the cathode may be taken as a narrow strip of width l_* which corresponds to the transit time of the outermost electron from the left in the scheme shown in Fig. 1, a. Here, the single-path region of the flux 1) (where $\delta=\rho$) on the trajectory of the outermost electron must be joined with the two-path region of the flux 2) in the free beam (where $\Sigma=0$). The formation conditions, however, may be selected such that $V\lesssim \varepsilon_*^3$ in the region 1); then, within an accuracy to ε^3, the narrow region 1) may be neglected, and the second condition in (1.15) may be taken into consideration. If the cathode is selected wider, we get a multiflux beam (scheme δ in Fig. 1). For $V\lesssim\varepsilon^3, l<l_*$ however, a zero velocity at the cathode for both fluxes is fulfilled within an accuracy to ε^3, and with the same accuracy, the real multiflux beam can be replaced by a two-flux beam with the condition (1.15) in the $0<l<l_*$ region of the cathode.

1.3°. The solution of the system (1.7)-(1.12) may be sought in the form of a power series in ε with the aid of successive approximations. Then (correct to ε^3), from (1.11) for the azimuthal velocity A we easily get

$$A=\Omega s+\Gamma+\varepsilon B_H s^2+\varepsilon^2(C_H s^3+D_H s^2), \qquad 2B_H\equiv(k-k_z)\Omega \tag{1.16}$$

$$2D_H\equiv\frac{\Gamma}{R^2}-\frac{(R\Gamma')'}{R}, \qquad 6C_H\equiv\left[2(k^2-kk_z+k_z^2)+\frac{1}{R^2}\right]\Omega-\frac{(R\Omega')'}{R}$$
$$\Omega=\Omega(l), \quad \Gamma=\Gamma(l), \quad \Omega+\varepsilon k_z\Gamma=(\eta/c)H_l^0 \tag{1.17}$$

where H_l^0 is the tangential component of the magnetic field intensity at the axis (s=0). However, the second approximation for the entire problem is too cumbersome. In order to simplify the calculation, the "smallness" under μ should be placed in front of every parameter in (1.7)-(1.12), and (1.16): $\mu(\omega)$ for ω, $\mu(V)$ for V, and so forth; for example from (1.7), (1.9) and (1.15), it follows that

$$\mu(v_s)=\varepsilon, \quad \mu(\Sigma)=\varepsilon, \quad \mu(\delta)=\varepsilon \tag{1.18}$$

By assuming a concrete μ, it is possible to identify simple cases of the mode of propagation of the beam; thus $\mu(\rho)=\varepsilon$, if $4\pi\eta\rho$ is of the order of $\varepsilon_*\Omega^2$.

2. First Approximation. Within an accuracy to ε^2, using (1.16) and (1.18), Eqs. (1.7)-(1.12) reduce to the two equations

$$w_s^2=2\eta\varphi-V^2-2(\Omega s+\Gamma)^2-2\varepsilon ksV^2-\varepsilon(k-k_z)(\Omega s+\Gamma)\Omega s^2 \tag{2.1}$$

$$\partial\varphi/\partial s=\left(E+4\pi I\int w_s^{-1}ds\right)[1+\varepsilon(k-k_z)s], \qquad E=E(l) \tag{2.2}$$

where $E(l)$ is the normal component of the field intensity at the axis. Written in the variables $\tau=\tau(s,l)$, l, Eqs. (2.1) and (2.2) take the form

$$s^{\bullet\bullet}+s=\eta(\partial\varphi/\partial s)\Omega^{-2}-\Gamma/\Omega-\varepsilon kV^2\Omega^{-2}-\varepsilon(k-k_z)(\Gamma/\Omega+{}^3/_2 s)s$$
$$\eta(\partial\varphi/\partial s)=(\alpha\tau\Omega^2+\eta E)[1+\varepsilon(k-k_z)s], \qquad \alpha\equiv4\pi\eta I\Omega^{-3} \tag{2.3}$$

$$s^\bullet\equiv\partial s/\partial\tau=w_s/\Omega, \qquad \tau'\equiv\partial\tau/\partial l=-(\partial s/\partial l)(\Omega/w_s) \tag{2.4}$$

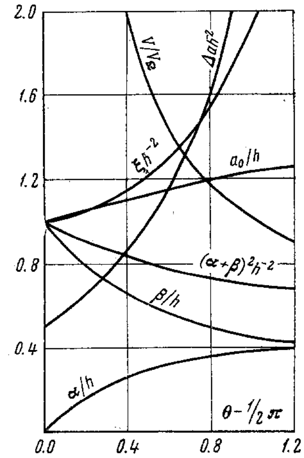

Fig. 2

As a result, we have one equation for $s(\tau, l)$

$$s^{\cdot\cdot} + s = \alpha\tau + \lambda + \varepsilon(k - k_z)\, s\,(\alpha\tau + \lambda - {}^3/_2 s), \quad \lambda\Omega^2 \equiv \eta E - \Omega\Gamma - \varepsilon k V^2 \tag{2.5}$$

The solution of (2.5) can be written within an accuracy of ε^2 as follows

$$\begin{aligned} s = {} & \alpha\tau + \beta s_\tau + \lambda(1 - c_\tau) + \varepsilon(k - k_z)\,[(\lambda^2 - \beta^2)(1 - c_\tau - {}^1/_2 s_\tau^2) \\ & + \alpha^2(1 - c_\tau - {}^1/_2\tau^2) - {}^1/_2\alpha\beta(\tau s_\tau - \tau^2 c_\tau) + {}^1/_2\alpha\lambda(\tau^2 s_\tau + \tau c_\tau - 2\tau \\ & + s_\tau) - 2\lambda^2(1 - c_\tau - {}^1/_2\tau s_\tau) + \beta\lambda(\tau c_\tau - s_\tau c_\tau), \quad \beta = \beta(l), \quad \lambda = \lambda(l) \end{aligned} \tag{2.6}$$

$$\begin{aligned} w_s / \Omega \equiv s^{\cdot} = {} & \alpha + \beta c_\tau + \lambda s_\tau + \varepsilon(k - k_z)\,[(\lambda^2 - \beta^2)(s_\tau - s_\tau c_\tau) - \alpha^2(\tau - s_\tau) \\ & - {}^1/_2\alpha\beta(s_\tau - \tau c_\tau + \tau^2 s_\tau) + {}^1/_2\alpha\lambda(\tau s_\tau + \tau^2 c_\tau + 2c_\tau - 2) - \lambda^2(s_\tau - \tau c_\tau) \\ & + \beta\lambda(c_\tau - 1 + 2s_\tau^2 - \tau s_\tau)] \end{aligned} \tag{2.7}$$

$$s_\tau \equiv \sin\tau, \quad c_\tau \equiv \cos\tau, \quad s_\theta \equiv \sin\theta, \quad c_\theta \equiv \cos\theta \tag{2.8}$$

2.1°. Let the axis be located symmetrically with respect to τ

$$\tau_+ = -\tau_- = \theta(l), \quad \tau(s = 0) = 0, \quad s_\pm = s(\pm\theta, l) \tag{2.9}$$

By placing the last conditions in (1.14) on (2.7), we get

$$\begin{aligned} & \alpha + \beta c_\theta = -\varepsilon(k - k_z)\,\lambda\,[{}^1/_2\alpha(\theta s_\theta + \theta^2 c_\theta + 2c_\theta - 2) + \beta(c_\theta - 1 + 2s_\theta^2 - \theta s_\theta)] \\ & \lambda s_\theta = -\varepsilon(k - k_z)\,[(\lambda^2 - \beta^2)(s_\theta - s_\theta c_\theta) - \alpha^2(\theta - s_\theta) - {}^1/_2\alpha\beta(s_\theta - \theta c_\theta + \theta^2 s_\theta) - \lambda^2(s_\theta - \theta c_\theta)] \end{aligned} \tag{2.10}$$

From (2.10) it follows that $\mu(\lambda) = \varepsilon$, provided $\mu(s_0) = 1$. In this case, conditions (1.14), (1.15) and (2.9), within an accuracy of ε^2, yield

$$\begin{gathered} s_\pm = \pm a_0 + \varepsilon(k - k_z)\,\Delta a, \quad a_0 \equiv \beta(s_\theta - \theta c_\theta) \\ \Delta a \equiv \beta^2\,[c_\theta^2 - {}^3/_2 c_\theta + {}^3/_2\theta c_\theta^2(1 - c_\theta)\,s_\theta^{-1} \\ - {}^1/_2\theta^2(c_\theta + 1)\,c_\theta + {}^1/_2 + {}^1/_2\theta s_\theta c_\theta] \\ \alpha = -\beta c_\theta \\ \xi = \beta^2({}^3/_2\theta c_\theta^2 s_\theta^{-1} - c_\theta^2 - {}^3/_2 c_\theta - {}^1/_2\theta^2 c_\theta + 1) \\ \beta^2\,[\theta(1 + 2c_\theta^2) - 3s_\theta c_\theta] = w_* / \Omega, \quad \alpha R V\theta = \eta J_+ \Omega^{-2} \\ \lambda \equiv \varepsilon\xi(k - k_z), \quad {}^1/_2\pi \leqslant \theta < \pi \end{gathered} \tag{2.11}$$

Figure 2 gives plots of dimensionless relations which depend only on θ

$$\begin{gathered} \frac{\beta}{h}, \quad \frac{a_0}{h}, \quad \frac{\Delta a}{h^2}, \quad \frac{V}{V_\Omega}, \quad h^2 \equiv \frac{2w_*}{\pi\Omega} \\ V_\Omega \equiv \frac{R\Omega^2}{\eta J_+} h \quad \left(\frac{\alpha}{h} + \frac{\beta}{h}\right)^2 \end{gathered} \tag{2.12}$$

The last quantity is proportional to the electron spinning energy $\mathscr{E}_\perp$. The quantity $2\eta\mathscr{E}_\perp(\Omega h)^{-2}$ decreases from 1 to 0.67 with an increase of the space charge ($\theta \to \pi$). The dependence of the beam parameters on an increasing magnetic-mirror field is clearly seen from (2.12), where Ω, correct to ε, is the cyclotron frequency (1.17).

2.2°. The case $\mu(s_\theta) \lesssim \varepsilon$ can be somewhat simplified by placing the axis on the inner boundary of the beam

$$\tau_- = s_- = 0, \quad \beta = -\alpha \tag{2.13}$$

In accordance with (2.6), (2.7) and (2.13), the conditions (1.14) and (1.15) yield

$$
\begin{aligned}
s_+ = \{&\alpha(\tau - s_\tau) + \lambda(1 - c_\tau) + \varepsilon^{1/2}(k - k_z)[\alpha^2(\tau s_\tau - \tau^2 + s_\tau^2 \\
&- \tau^2 c_\tau) + \alpha\lambda(s_\tau - \tau c_\tau + \tau^2 s_\tau + 2 s_\tau c_\tau - 2\tau) + \lambda^2(2c_\tau - 2 - s_\tau^2 + 2\tau s_\tau)]\}_+ \\
\{s'\}_+ = \{&\alpha(1 - c_\tau) + \lambda s_\tau + \varepsilon^{1/2}(k - k_z)[\alpha^2(s_\tau - 2\tau - \tau c_\tau + \tau^2 s_\tau + 2 s_\tau c_\tau) \\
&+ \alpha\lambda(3\tau s_\tau + \tau^2 c_\tau - 4 s_\tau^2) + \lambda^2(2\tau c_\tau - 2 s_\tau c_\tau)]\}_+
\end{aligned}
\tag{2.14}
$$

$$
\begin{aligned}
w_+ = \Omega\{&\alpha^2(3/2\,\tau - 2s_\tau + 1/2\, s_\tau c_\tau) + \alpha\lambda(1 - c_\tau)^2 + 1/2\,\lambda^2(\tau - s_\tau c_\tau) + \varepsilon(k - k_z)[\alpha^3(3\tau s_\tau \\
&- \tau^2 c_\tau + 2c_\tau - 2 - 1/2\,\tau^2 s_\tau^2 + 1/2\, s_\tau^2 + 2/3\, c_\tau^2 - 2/3 - 1/2\,\tau^2) + \alpha^2\lambda(\tau^2 s_\tau - \tau^2 s_\tau c_\tau + \tau c_\tau - s_\tau \\
&+ s_\tau c_\tau - 2\tau + 2 s_\tau^3 + \tau c_\tau^2) + \lambda^2\alpha(2\tau s_\tau - 2\tau s_\tau c_\tau + 1/2\,\tau^2 s_\tau^2 - 4 + 6c_\tau - 2c_\tau^3) \\
&+ \lambda^3(\tau s_\tau^2 - 1/2\,\tau + 1/2\, s_\tau c_\tau - 2/3\, s_\tau^3)]\}_+ = w_*
\end{aligned}
\tag{2.15}
$$

$$
J_+ = 1/2\,(\Omega^2/\eta)\,\alpha V\{\tau + 2\varepsilon k[\alpha(1/2\,\tau^2 - c_\tau + 1) + \lambda(\tau - s_\tau)]\}_+
\tag{2.16}
$$

For small x, from (2.14)-(2.16), it is easy to obtain

$$
\begin{gathered}
\tau_+ \equiv 2(\pi - \varepsilon x), \qquad x = \pi(k - k_z)[\lambda - 3/2\,\alpha^2/\lambda + \pi\alpha] \\
3\alpha^2 + \lambda^2 = w_*/(\pi\Omega) + \varepsilon(k - k_z)(\lambda^3 + 6\pi\alpha^3) \\
s_+ = \pi\alpha - \varepsilon\pi(k - k_z)(4\pi\alpha^2 + 3\alpha\lambda), \qquad J_+ = (\Omega^2/\eta)\,\alpha V\{\pi + \varepsilon[2\pi k(\lambda + \pi\alpha) - x]\}
\end{gathered}
\tag{2.17}
$$

It should be noted that at the singular point at the boundary

$$
\partial\Phi/\partial s = \Phi = 0 \qquad (s'' = s' = 0)
\tag{2.18}
$$

there may occur a branching of the beam boundary and, therefore, of the beam itself. If (2.18) is fulfilled over a finite segment, for example at the lower boundary, then a two-stream mode is possible, where the solution (2.14)-(2.16) for $\tau < 0$ describes the second beam when it adjoins the first beam from below along the boundary s=0. In this region we have $\lambda = 0$, while a current across the boundary is possible ($\Sigma \neq 0$).

3. Small Space Charge, Second Approximation. Setting $\mu(\mathrm{I}) = \varepsilon$, in correspondence with (1.9) and (1.8), we have

$$
\mu(v_s) = \varepsilon, \qquad \mu(\Sigma) = \mu(\delta) = \varepsilon^2, \qquad \mu(\rho) = \mu(\alpha) = \varepsilon
\tag{3.1}
$$

From (1.9), (1.11), with the aid of (2.2) and (2.4), it is easy to get

$$
[1 - \varepsilon(k - k_z)s - \varepsilon^2 k k_z s^2]\,\eta\,\frac{\partial\varphi}{\partial s} = \eta E + \varepsilon\alpha\tau\Omega^2 - \varepsilon^2(\eta/R)\int(R\varphi')'\,ds
\tag{3.2}
$$

3.1°. The zero approximation which derives in the case of (3.1) from (1.7)-(1.12), (1.16), (2.4), and (2.9), (1.14) has the form

$$
s = \beta s_\tau, \quad w_s = \Omega\beta c_\tau, \quad w = 1/2\,\Omega\beta^2(\tau + s_\tau c_\tau), \quad \theta = 1/2\,\pi, \quad \beta = \beta(l)
\tag{3.3}
$$

Using (3.3), (1.7)-(1.12), (1.16) and (2.4), together with the conditions (1.14) and (1.15), it is not difficult to calculate the following values;

$$
\begin{gathered}
\tau' = -k_\beta s\Omega/w_s, \quad w_l = -w_s k_\beta s, \quad v_s = V k_\beta s, \quad k_\beta \equiv \beta'/\beta \\
v_l = V + \varepsilon^2\, 1/2\,(V k_\beta)' s^2
\end{gathered}
\tag{3.4}
$$

$$
(R\varphi')' = (RU')' + (s/\eta)[(\Omega\Gamma)'R]', \quad 2\eta U \equiv \Gamma^2 + V^2 + \Omega^2\beta^2
\tag{3.5}
$$

$$J = 2\pi RI\,(V/\Omega)\,(\tau - \tau_-), \qquad Q = \Sigma R\,(V/\Omega)\,(T - {}^1/_2\,T^2)\,(\tau_+ - \tau_-)$$
$$- {}^1/_2\,IRk_\beta\,(s^2 - s_-^2), \qquad T \equiv (\tau - \tau_-)/(\tau_+ - \tau_-)$$
$$Rv\sigma\rho w_s = IR - \varepsilon^2\,[(R\Sigma V/\Omega)'\,(T - {}^1/_2T^2)\,(\tau_+ - \tau_-) - {}^1/_2\,(RIk_\beta)'\,s^2 + {}^1/_2\,(RIk_\beta s_-^2)'],$$
$$\delta w_s = \Sigma\,(1 - T) \tag{3.6}$$
$$w_s^2 = 2\eta\varphi - (\Omega s + \Gamma)^2 - V^2 - \varepsilon 2s\,[kV^2 + (\Omega s + \Gamma)\,B_H s] - \varepsilon^2\,\{[V^2\,(k_\beta^2 + 3k^2)$$
$$+ V\,(Vk_\beta)']\,s^2 + 2\,(\Omega s + \Gamma)\,(C_H s^3 + D_H s^2) + B_H^2 s^4 + w_l^2\} \tag{3.7}$$

Differentiation of (3.7) with respect to s leads with allowance for (2.4), (3.2), (3.5) and (1.16) to an equation for the function $s(\tau, l)$

$$s^{\cdot\cdot} + s = \varepsilon\,[\alpha\tau + (\xi - {}^3/_2 s^2)\,(k - k_z)] + \varepsilon^2\,(k - k_z)\,s\alpha\tau - \varepsilon^2\,(A_w s + B_w s^2 + C_w s^3)$$
$$\eta E - \Omega\Gamma - \varepsilon k V^2 \equiv \varepsilon\xi\,(k - k_z)\,\Omega^2$$
$$\Omega^2 A_w \equiv (\eta/R)\,(RU')' - (k - k_z)^2\,\xi\Omega^2 + V^2\,(k_\beta^2 + 2k^2 + kk_z + k_\beta')$$
$$+ VV'k_\beta + \Gamma^2 R^{-2} - (\Gamma/R)\,(R\Gamma')' + \Omega^2 k^2\beta^2, \qquad k_z \equiv Z'/R$$
$$\Omega^2 B_w = 2\Omega\Gamma R^{-2} - \Omega\Gamma'' - \Omega\Gamma' k_R + \Omega'\Gamma', \qquad k_R \equiv R'/R$$
$$C_w \equiv {}^1/_6\,(11k^2 - 14kk_z + 11k_z^2) - {}^2/_3 R^{-2} - 2k_\beta^2 - {}^2/_3\,[\Omega''/\Omega + (\Omega'/\Omega)\,k_R] \tag{3.8}$$

3.2°. The solution of (3.8), correct to ε^2, has the form

$$s = \beta s_\tau + \varepsilon\,\{\alpha\,(\tau - s_\tau) + (k - k_z)\,[(\xi - \beta^2)\,(1 - c_\tau) + {}^1/_2\beta^2 s_\tau^2] \tag{3.9}$$

Analysis of the first approximation of (3.9) for the conditions (1.14) shows that it is permissible to set

$$\theta = {}^1/_2\pi + \varepsilon x, \quad \mu\,(x) = 1, \quad (\xi - \beta^2)\,(k - k_z) = \varepsilon\zeta\beta, \quad \mu\,(\zeta) = 1 \tag{3.10}$$

Solving (3.8) with allowance for (3.9) and (3.10), one obtains without difficulty

$$s = \beta s_\tau + \varepsilon\,[\alpha\,(\tau - s_\tau) + {}^1/_2\,(k - k_z)\,\beta^2 s_\tau^2] + \varepsilon^2\beta\,[\alpha\,(k - k_z)\,(2 - 2c_\tau - s_\tau^2$$
$$- {}^1/_2\,\tau s_\tau + {}^1/_2\,\tau^2 c_\tau) + \zeta\,(1 - c_\tau) - {}^1/_2\,A_w\,(s_\tau - \tau c_\tau) - {}^2/_3\,B_w\beta\,(1 - c_\tau - {}^1/_2\,s_\tau^2)$$
$$- {}^1/_4\,D_w\beta^2\,(s_\tau^3 - {}^3/_2\,\tau c_\tau + {}^3/_2\,s_\tau c_\tau^2)] \tag{3.11}$$
$$s^{\cdot} = \beta c_\tau + \varepsilon\,[\alpha\,(1 - c_\tau) + (k - k_z)\,\beta^2 s_\tau c_\tau] + \varepsilon^2\beta\,[\alpha\,(k - k_z)\,({}^3/_2\,s_\tau - 2s_\tau c_\tau$$
$$+ {}^1/_2\,\tau c_\tau - {}^1/_2\,\tau^2 s_\tau) + \zeta s_\tau - {}^1/_2\,A_w\tau s_\tau - {}^2/_3\,B_w\beta\,(s_\tau - s_\tau c_\tau) - {}^3/_8\,D_w\beta^2\,s_\tau\,(\tau - s_\tau c_\tau)$$
$$D_w \equiv c_w + {}^3/_2\,(k - k_z)^2 \tag{3.12}$$

The conditions (1.14), (2.9), and (3.10)-(3.14) make it possible to obtain

$$s_\pm = \pm\,\{\beta + \varepsilon\,({}^1/_2\,\pi - 1)\,\alpha - \varepsilon^2\,{}^1/_2\beta\,(A_w + {}^1/_2\,D_w\beta^2 - \alpha^2\beta^{-2})\}$$
$$+ {}^1/_2\,\varepsilon\,(k - k_z)\,\beta^2 + \varepsilon^2\beta\,[({}^1/_8\,\pi^2 - {}^1/_4\,\pi + {}^1/_2)\,\alpha\,(k - k_z) + {}^1/_3\,B_w\beta] \tag{3.13}$$
$$x = \alpha/\beta\,(1 + \varepsilon\alpha/\beta) - \varepsilon{}^1/_4\,\pi\,(A_w + {}^3/_4\,D_w\,\beta^2) \tag{3.14}$$
$$\zeta = {}^1/_2\,({}^1/_4\,\pi^2 - 1)\,\alpha\,(k - k_z) + {}^2/_3\,B_w\beta \tag{3.15}$$

Further, from (1.8), (3.10), (3.11) and (3.14) it is possible to calculate the invariant ω_+, and from (1.10), (3.4), (3.6), (3.11) and (3.15) the current J_+

$$w_*/\Omega = {}^1/_2\,\pi\beta^2 + \varepsilon\,(4 - \pi)\,\alpha\beta + \varepsilon^2\,[({}^3/_2\,\pi - 4)\,\alpha^2 + {}^1/_8\,\pi\beta^4\,(k - k_z)^2$$
$$- {}^1/_4\,\pi\beta^2\,(A_w + {}^3/_8\,D_w\beta^2)] \tag{3.16}$$

$$J_+ = 2\pi^2 RI\,(V/\Omega)\,\{1 + \varepsilon\,(2/\pi)\,x + \varepsilon^2\beta^2\,[2k^2 - {}^1/_2\,kk_z - k_\beta^2 + {}^1/_4\,k_\beta\,(V'/V - I'/I$$
$$- R'/R)]\} + \varepsilon^2\pi\,[{}^1/_2\,\Sigma Rk_\beta\beta^2 - {}^2/_3\,\pi^2\,(V/\Omega)\,(\Sigma RV/\Omega)'] \tag{3.17}$$

The formulas obtained are suitable for calculating all the parameters of the beam with an accuracy to ε^2, provided the position of the axis is known.

From (3.8), (3.10), (3.2), (3.5) and (1.16), one obtains the relation

$$\eta(\Psi_R Z' - \Psi_Z R') = \varepsilon k 2\eta\Psi_{s=0} - \varepsilon k_z \beta^2\Omega^2 + \varepsilon^2\zeta\beta\Omega^2$$
$$2\eta\Psi \equiv 2\eta\varphi - A^2, \quad \Psi_R \equiv (\partial\Psi/\partial r)_{s=0}, \quad \Psi_Z \equiv (\partial\Psi/\partial z)_{s=0}, \quad \Omega = (\partial A/\partial s)_{s=0} \tag{3.18}$$

which constitutes the equation for the axial surface. In the zero approximation, the axis obviously runs along the lines of force of potential Ψ, without allowance for the space charge. In the following approximations, one must take into account the space charge field in Ψ and the terms in the right side of (3.18), as defined by (3.14)-(3.17). To prevent the beam from being reflected from the mirror, and to prevent its branching, it is necessary to eliminate both the regions where $2\eta\Psi(R, Z) < \Omega^2\beta^2$, and the singular points of the Ψ field at the axis, in which $\Psi_R = \Psi_Z = 0$.

3.3°. In the absence of a space charge, the field can be represented in a form analogous to (1.16)

$$\varphi = U + Es + \varepsilon B_H s^2 + \varepsilon^2(C_E s^3 + D_E s^2), \quad U = U(l), \quad E = E(l)$$
$$2B_E \equiv (k - k_z)E, \quad 2D_E \equiv -(RU')'/R$$
$$6C_E \equiv 2(k^2 - kk_z + k_z^2)E - (RE')'/R \tag{3.19}$$

Changing to the variable τ in accordance with (2.4) is unnecessary. Equations (3.3), (3.4), (3.6) and (3.7) remain valid, provided β and τ are treated as parameters, and the following substitutions are introduced in all the approximations

$$s \equiv \beta s_\tau, \quad s_\pm = \pm\beta, \quad \tau_\pm = \pm{}^1/_2\pi, \quad \beta = \beta(l) \tag{3.20}$$

From (3.7), (3.4), (3.19), (1.14) and (3.20) it follows that:

$$w_s = \Omega(\beta^2 - s^2)^{1/2}\{1 + {}^1/_2\varepsilon(k - k_z)s + {}^1/_2\varepsilon^2(A_\Phi + B_\Phi s + C_\Phi s^2)\} \tag{3.21}$$

$$A_\Phi\Omega^2 = V^2(k_\beta^2 + 2k^2 + kk_z + k_\beta') + VV'k_\beta + (2\Omega C_H - B_H^2)\beta^2 + 2\Gamma D_H$$
$$- 2\eta D_E, \quad B_\Phi\Omega^2 \equiv 2\Omega D_H + 2\Gamma C_H - 2\eta C_E, \quad C_\Phi \equiv 2C_H/\Omega - k_\beta^2$$
$$V^2 = 2\eta U - \Gamma^2 - \Omega^2\beta^2 - \varepsilon^2 A_\Phi\beta^2\Omega^2 \tag{3.22}$$

$$\eta E - \Omega\Gamma = \varepsilon k(2\eta U - \Gamma^2) - \varepsilon^{1/2}(k + k_z)\Omega^2\beta^2 + {}^1/_2\varepsilon^2 B_w\Omega^2\beta^2 \tag{3.23}$$

where, by analogy to (3.18), Eq. (3.23) is here the equation for the axis that passes through the center of the beam. The beam half-width β is determined by the invariant ω_*

$$\beta^2 = h^2\{1 - {}^1/_2\varepsilon^2(A_\Phi + {}^1/_4 C_\Phi h^2)\}, \quad h^2 \equiv (2/\pi)(w_*/\Omega) \tag{3.24}$$

The current of the free beam ($\Sigma = 0$) is determined from (3.6), (3.20) and (3.21) as follows

$$J_+ = 2\pi^2 R(V/\Omega)\{1 - {}^1/_2\varepsilon^2 A_\Phi + \varepsilon^2\beta^2[k^2 + {}^1/_2 kk_z - k_\beta^2 + {}^1/_8(k - k_z)^2 - {}^1/_4 C_\Phi$$
$$+ {}^1/_4 k_\beta(V'/V - I'/I - R'/R)]\} \tag{3.25}$$

For the mean rate of charge transport (ratio of beam current to charge per unit length) in the first approximation, from (1.16), (3.6), (3.21) and (3.23), one gets

$$\langle v_l\rangle = V, \quad \langle A\rangle = \eta E/\Omega - \varepsilon\,{}^1/_2 k(2V^2 + \beta^2\Omega^2)/\Omega, \quad \eta(\partial H/\partial s)_{s=0} = \varepsilon k\Omega$$

This result correlates well with Eq. (25.46) in [1] for the velocity of motion of the Larmor center, under the assumption $\mu(E) = \varepsilon$ employed in [1].

4. Large Space Charge. With the aid of solution (2.14), the following regime can be identified;

$$\tau_+ = 2\pi + \sqrt{\varepsilon}(p \pm q), \quad \mu(\lambda) = \sqrt{\varepsilon} \quad (\tau_- = s_- = 0) \tag{4.1}$$

$$
\begin{gathered}
s = \alpha(\tau - s_\tau) + \sqrt{\varepsilon}\lambda\,[(1 - c_\tau) + \varepsilon\, {}^1/_2 (k - k_z)\,\alpha\,(s_\tau - \tau c_\tau + \tau^2 s_\tau - 2\tau + 2 s_\tau c_\tau)] \\
+ \varepsilon\, {}^1/_2 (k - k_z)\,\alpha^2 (\tau s_\tau - \tau^2 + s_\tau{}^2 - \tau^2 c_\tau) + \varepsilon^2 s_2 \\
s^{\cdot} = \alpha(1 - c_\tau) + \sqrt{\varepsilon}\lambda\,[s_\tau + \varepsilon\, {}^1/_2 (k - k_z)\,\alpha\,(3\tau s_\tau + \tau^2 c_\tau - 4 s_\tau{}^2)] \\
+ \varepsilon\, {}^1/_2 (k - k_z)\,\alpha^2 (s_\tau - 2\tau - \tau c_\tau + \tau^2 s_\tau + 2 s_\tau c_\tau) + \varepsilon^2 s_2^{\cdot}
\end{gathered} \tag{4.2}
$$

This regime lends itself to computation with an accuracy to $\varepsilon^{5/2}$. According to (4.1) and (4.2), conditions (1.14) define two roots of τ_+

$$
\begin{gathered}
p \doteq -\varkappa - \sqrt{\varepsilon}\,2\pi^2 (k - k_z)\,\alpha + \varepsilon\varkappa\,[{}^2/_3\,\varkappa^2 + \pi(k - k_z)\,\alpha] \\
q^2 = \varkappa^2 + 6\pi(k - k_z)\,\alpha - \varepsilon\,[\pi^2 (30 + 4\pi^2)(k - k_z)^2 \alpha^2 + {}^4/_3\,\varkappa^4 \\
+ 6\pi\varkappa^2 (k - k_z)\,\alpha] - \varepsilon\,(2/\alpha)\, s_2^{\cdot}(2\pi), \qquad \varkappa \equiv \lambda/\alpha
\end{gathered} \tag{4.3}
$$

The invariant ω_* is determined from (2.15) with allowance for (4.1), as follows

$$
w_* = \pi\Omega\,[3\alpha^2 + \varepsilon\lambda^2 - \varepsilon 6\pi (k - k_z)\,\alpha^3] + \varepsilon^2 w_2 (2\pi) \tag{4.4}
$$

The beam current is calculated from (1.10), (3.4), (3.6) and (4.2).

$$
\begin{gathered}
J_+ = 2\pi^2 IR\,(V/\Omega)\,\{2 + (\sqrt{\varepsilon}/\pi)(p \pm q)(1 + \varepsilon 4\pi k\alpha) + \varepsilon 4k\,(\pi\alpha + \varepsilon^{1/2}\lambda) \\
+ \varepsilon^2 ({}^8/_3\,\pi^2 + 5)\,\alpha^2 [2k^2 + kk_z + k_\alpha' + {}^1/_2\, k_\alpha (V'/V + I'/I + R'/R)] \\
- \varepsilon^2 2\pi\,[{}^4/_3\,\pi^2 (V/\Omega)(R\Sigma V/\Omega)' + ({}^2/_3\,\pi^2 + {}^5/_4)\, R\Sigma k_\alpha \alpha^2]
\end{gathered}
$$

The corrections s_2, $s_2^{\cdot}$, ω_2 in (4.2)-(4.4) are defined by the second approximation, which will be obtained below.

4.1°. Within the framework of the zero approximation, one may use (3.4), (3.6) and (3.7), if in accordance with (4.1) and (4.2), 2π is substituted for τ_+, and $k_\alpha = \alpha'/\alpha$ for k_β. The field is determined from (1.11) with the aid of (2.4), (4.2), (3.4), (3.6) and (3.7)

$$
\begin{gathered}
\eta\,(\partial\varphi/\partial s) = (\alpha\tau\Omega^2 + \eta E)\,[1 + \varepsilon (k - k_z)\, s + \varepsilon^2 (k^2 - kk_z + k_z^2)\, s^2] \\
+ \varepsilon^2 \alpha\Omega^2 \{{}^1/_2\,\alpha^2 (RIk_\alpha)'(RI)^{-1} ({}^1/_3\,\tau^3 + 2\tau c_\tau - 2 s_\tau + {}^1/_2\,\tau - {}^1/_2\, s_\tau c_\tau) \\
- {}^1/_2 (R\Sigma V/\Omega)'(RI)^{-1} [\tau^2 - \tau^3 (6\pi)^{-1}]\} - \varepsilon^2 (\eta/R)\int (R\varphi')'\, ds \\
\eta\int (R\varphi')'\, ds = {}^1/_3 (R\Omega\Omega')'\, s^3 + {}^1/_2 [(\Omega\Gamma)' R]'\, s^2 + {}^1/_2 [(V^2 + \Gamma^2)' R]'\, s \\
+ {}^1/_2\,\Omega\alpha^3 [(R\Omega')' ({}^9/_4\,\tau + {}^7/_4\, s_\tau c_\tau - 3 s_\tau - \tau c_\tau - {}^1/_2\,\tau s_\tau{}^2 + {}^2/_3\, s_\tau{}^3) \\
- R\Omega' k_\alpha (s_\tau - \tau c_\tau + \tau^2 s_\tau - {}^5/_2\,\tau s_\tau{}^2 - {}^1/_4\,\tau + {}^1/_4\, s_\tau c_\tau + {}^4/_3\, s_\tau{}^3)]
\end{gathered} \tag{4.5}
$$

Differentiation of (3.7) with respect to s with allowance for (2.4), (4.1) and (4.5), leads to the equation

$$
\begin{gathered}
s_2^{\cdot\cdot} + s_2 = {}^1/_2 (k - k_z)\,\lambda^2 (4c_\tau - 3c_\tau{}^2 - 1) + B_\rho \alpha^2 (2\tau s_\tau - s_\tau{}^2) \\
- \tau^2 \alpha\,[B_\rho \alpha + {}^1/_2\,\alpha^2 \Lambda_\Sigma] + \alpha^3 \{{}^1/_2 (k - k_z)^2 (2\tau^3 c_\tau - 3\tau^2 s_\tau c_\tau) \\
- \tau\,[2k_\alpha{}^2 + {}^9/_8\,\Lambda_R + {}^1/_8\,\Lambda_\Omega - {}^1/_4\,\Lambda_I + A_\rho \alpha^{-2}] + \tau^3 [(k - k_z)^2 \\
+ \Lambda_\Sigma (12\pi)^{-1} + {}^1/_6\,\Lambda_I - C_\rho] + s_\tau [2k_\alpha{}^2 + {}^3/_2\,\Lambda_R + {}^1/_2\,\Lambda_\Omega - \Lambda_I + A_\rho \alpha^{-2}] \\
+ \tau c_\tau [2k_\alpha{}^2 + {}^1/_2\,\Lambda_R - {}^1/_2\,\Lambda_\Omega + \Lambda_I] - s_\tau c_\tau [2k_\alpha{}^2 + {}^7/_8\,\Lambda_R - {}^1/_8\,\Lambda_\Omega + {}^1/_4\,\Lambda_I] \\
+ \tau s_\tau{}^2 [3k_\alpha{}^2 + {}^1/_4\,\Lambda_R - {}^5/_4\,\Lambda_\Omega - 3C_\rho - {}^1/_2 (3k^2 - 2kk_z + 3k_z^2)] + \tau^2 s_\tau [{}^1/_2\,\Lambda_\Omega \\
+ 3C_\rho - k_\alpha{}^2 - {}^1/_2\,(3k + 3k_z^2 - 8kk_z)] + s_\tau{}^3 [-2k_\alpha{}^2 - {}^1/_3\,\Lambda_R + {}^2/_3\,\Lambda_\Omega + C_\rho \\
+ {}^1/_2 (5k^2 - 8kk_z + 5k_z^2)]\} \\
\Lambda_\Sigma \equiv (R\Sigma V/\Omega)'(RI\alpha^2)^{-1}, \quad \Lambda_R \equiv (R\Omega')'(R\Omega)^{-1}, \quad \Lambda_\Omega \equiv (\Omega'/\Omega)\, k_\alpha \\
\Lambda_I \equiv (RIk_\alpha)'(RI)^{-1}, \quad k_\alpha \equiv \alpha'/\alpha, \quad k_z \equiv Z'/R, \quad k_R \equiv R'/R \\
A_\rho \Omega^2 \equiv V^2 (2k^2 + kk_z + k_\alpha{}^2 + k_\alpha') + VV'(k_\alpha + k_R) + (VV')' + \Gamma^2 R^{-2} + (\Gamma')^2 \\
B_\rho \Omega \equiv 2\Gamma R^{-2} + (\Omega'/\Omega)\,\Gamma - \Gamma'' + \Gamma' k_R \\
6C_\rho \equiv 5k^2 - 8kk_z + 5k_z^2 + 4R^{-2} + 2\,[(\Omega'/\Omega)^2 - (\Omega'/\Omega)\, k_R + \Omega''/\Omega]
\end{gathered} \tag{4.6}
$$

4.2°. The solution of Eq. (4.6) can be conveniently written in the form

$$
\begin{gathered}
s_2 = {}^1/_2 (k - k_z)\,\lambda^2 (2\tau s_\tau + 2c_\tau - 2 - s_\tau{}^2) + B_\rho \alpha^2 ({}^1/_2\,\tau s_\tau - {}^1/_2\,\tau^2 c_\tau - {}^2/_3 + {}^2/_3\, c_\tau + {}^1/_3\, s_\tau{}^2) \\
- (\tau^2 - 2c_\tau + 2)\,\alpha F[\tau^2] + \alpha^3 \{(\tau^3 + 6 s_\tau - 6\tau)\, F[\tau^3] - (\tau - s_\tau)\, F[\tau] \\
+ {}^1/_2 (s_\tau - \tau c_\tau)\, F[s_\tau] + {}^1/_4 (\tau^2 s_\tau + \tau c_\tau - s_\tau)\, F[\tau c_\tau] - {}^1/_3 (s_\tau - s_\tau c_\tau)\, F[s_\tau c_\tau] \\
+ {}^1/_3 (2\tau - {}^2/_3\, s_\tau - \tau s_\tau{}^2 - {}^4/_3\, s_\tau c_\tau)\, F[\tau s_\tau{}^2] + {}^1/_4 (\tau^2 s_\tau - s_\tau + \tau c_\tau - {}^2/_3\,\tau^3 c_\tau)\, F[\tau^2 s_\tau] \\
+ {}^1/_4 (s_\tau{}^3 - {}^3/_2\,\tau c_\tau + {}^3/_2\, s_\tau c_\tau{}^2)\, F[s_\tau{}^3] + {}^1/_2 (k - k_z)^2 [{}^1/_4 (3 - 3\tau^2 + \tau^4)\, s_\tau \\
+ {}^1/_4 (2\tau^3 - 3\tau)\, c_\tau + {}^4/_3 (\tau - 2\tau s_\tau{}^2 - s_\tau c_\tau) + \tau^2 s_\tau c_\tau + {}^{14}/_9 (s_\tau - s_\tau c_\tau)]\}
\end{gathered} \tag{4.7}
$$

Here, F [f] denotes the coefficients in square brackets in Eq. (4.6), which are situated behind the functions f. Solution (4.7) permits determination of the corrections in (4.3), (4.4)

$$
\begin{gathered}
s'_2(2\pi) = \pi\{2(k-k_z)\lambda^2 - \alpha^2 B_\rho - 4\alpha F[\tau^2]\} + \pi^2\alpha^3\{F[\tau c_\tau] + 12F[\tau^3] \\
- F[\tau^2 s_\tau] + (k-k_z)^2(4\pi^2 + 3/2)\} \\
w_2(2\pi)/\Omega = -2\pi^2\alpha^2\{B_\rho\alpha + 4F[\tau^2]\} + \pi\alpha^4\{2/3\,F[s_\tau c_\tau] - 6F[\tau] + 4(4\pi^2 - 15)F[\tau^3] \\
- 3/2\,F[s_\tau] - (2/3\,\pi - 1)F[\tau c_\tau] + 20/9\,F[\tau s_\tau^2] - 15/16\,F[s_\tau^3] - (4/3\,\pi^2 - 5/4)F[\tau^2 s_\tau] \\
+ 1/4\,[62\pi^2 - 550/18](k-k_z)^2\}
\end{gathered}
$$

From (4.1), (4.2) and (4.7) an expression for the upper boundary follows

$$
\begin{gathered}
s_+ = 2\pi\alpha - \varepsilon 4\pi^2(k-k_z)\alpha^2 + \varepsilon^{3/2}[1/2\,\alpha x^3 - 3\pi(k-k_z)\alpha(\lambda + \alpha x) + 1/2\,\lambda x^2] \\
+ \varepsilon^2\pi^2\alpha\{(k-k_z)x(2\lambda + \alpha x) - 2\alpha B_\rho - 4F[\tau^2]\} + \varepsilon^2\alpha^3\pi\{(8\pi^2 - 12)F[\tau^3] - 2F[\tau] \\
- F[s_\tau] + 1/2\,F[\tau c_\tau] + 4/3\,F[\tau s_\tau^2] - (4/3\,\pi^2 - 1/2)F[\tau^2 s_\tau] - 3/4\,F[s_\tau^3] \\
+ (k-k_z)^2(2\pi^2 + 7/12)]
\end{gathered}
$$

If the axis is selected from (2.9), the example we have examined will be relatively simpler and the obtainable accuracy will be higher. However, the selection of the axis as performed in the present analysis leads to simple expressions for the potential φ_- and the magnetic field intensity E_- at the lower boundary:

$$E_- = E, \qquad 2\eta\varphi_- = \Gamma^2 + V^2$$

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